

# Performance of a small wind energy generator under different configurations and settings

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## Abstract

Market of small wind turbine generators (SWTG) has been continuously growing during recent years. In this context, the offer for components for this technology has been also increased. The main components of SWTG are the rotor, the generator and the wind energy conversion system (WECS), typically formed by a rectifier plus an inverter. The settings of WECS affects to the SWTG behavior, especially those related with maximum and minimum voltages and the way to establish the inverter power curve. In this paper a method to calculate the inverter power curve of a STWG is presented, the impact of different types of curves is also evaluated.

## Keywords

Wind power generation, AC-AC power conversion, permanent magnet generators

## Introduction

Due to continued market growth of small wind turbine generators (SWTG) in recent years, the study of their behavior to improve their performance is an important issue. Nowadays, the design and manufacture of these SWTG's tends to the use of permanent magnet synchronous generators (PMSG). Thus in recent years, studies about them have been done, mainly about control and design methods [1,2,3,10].

PMSG's require wind energy conversion systems (WECS) that are used as interface between the generator (variable voltage and frequency) and the grid (fixed voltage and frequency).

There are several WECS configurations: with only diode rectifier and a inverter [1,2], with a DC/DC converter between rectifier and inverter [4,5,6], with a controlled rectifier [7,8], and with a controlled rectifier and a DC/DC converter [9]. However, the typical one is that composed by a rectifier, an inverter and dump load resistors is shown in Fig. 1. Dump loads are used to limit the AC voltage in the generator side or the DC voltage; usually they are connected or dimmed when a preset voltage value is achieved. In this configuration, most of the setting values are implemented in the inverter, and the power curve values are one of the most important settings, because they are directly related to the energy production.

In this paper, a study based on SWTG's with PMSG is presented. This paper focuses on the study of the influence of PMSG parameters and WECS configuration in energy production.

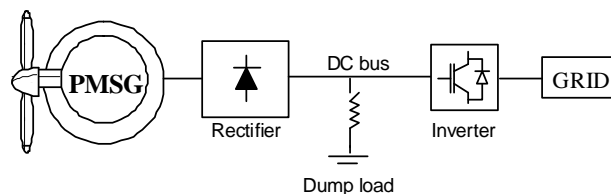


Fig. 1: General scheme of a wind turbine

## System description and modeling

The power configuration of the analyzed system is shown in Fig. 1. The wind-turbine used in this work is a direct-drive system. The generator is connected to the network through a rectifier and an inverter (WECS). The torque produced by the wind-turbine is used as the input torque to the PMSG and the voltage produced by the PMSG is rectified and passed through an inverter to the network. A maximum-power strategy is necessary to extract the maximum-power out of the system. In this case, the maximum power strategy is implemented by means of the inverter power curve.

The main parameters of the simulated SWTG are summarized in TABLE I.

TABLE I. SWTG main parameters

ROTOR	
Type	Up-wind / fixed pitch
Number of blades	3
Diameter	2.8 m
GENERATOR	
Type	PMSG
Pole pairs	12
Nominal voltage	230 V
Nominal speed	220 rpm
Power	6600 W
Steady state resistance	1.63 $\Omega$
Steady state inductance	22.46 mH
No-load voltage at nominal speed	190.3 V
RECTIFIER	
Type	Full wave / diode
Power	7,2 kW
Efficiency	99,5%
Max. input voltage	400 V <sub>AC</sub>
Dump load voltage	530 V <sub>DC</sub>
INVERTER	
Efficiency	97%
Power	6 kW
Input voltage	50 – 560 V <sub>DC</sub>

### Wind power extraction

The mechanical power ( $P_m$ ) captured by the rotor can be calculated by means of the following equation:

$$P_m = \frac{1}{2} \rho A C_p(\lambda) v^3 \eta_{mec} \quad (1)$$

Where  $\rho$  is the air density (typ. 1,225 kg/m<sup>3</sup>),  $A$  is the area covered by blades' rotation,  $C_p$  is the power coefficient and  $v$  is the wind speed (m/s) and  $\eta_{mec}$  is the mechanical performance. The power coefficient depends on the tip speed ratio  $\lambda$  calculated with the following expression:

$$\lambda = \Omega R / v \quad (2)$$

Where  $\Omega$  is the rotational speed (1/s) and  $R$  is the blade radius.

The relationship between  $C_p$  and  $\lambda$  is non-linear and for fixed pitch rotor can be modeled with following equation [11]:

$$C_p = \frac{1}{2} \left( \frac{RC_f}{\lambda} - k_1 \right) e^{-k_2 \frac{RC_f}{\lambda}} \quad (3)$$

Where  $C_f$  is a blade design constant, and  $k_1$  and  $k_2$  are constant parameters. These parameters have been calculated from real data rotor behavior, and the result is shown in Fig. 2.

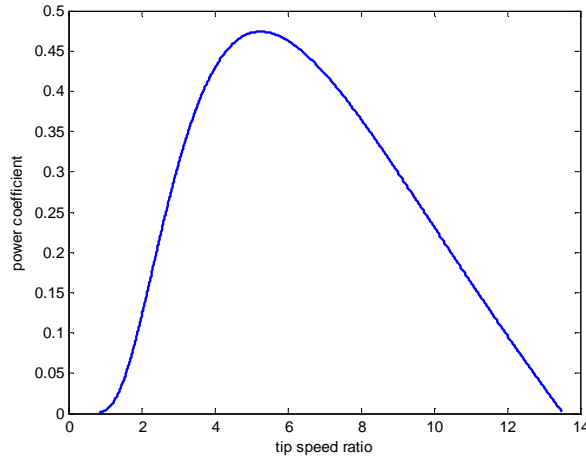


Fig. 2: Power coefficient curve

For each wind speed, there exists an optimum tip speed ratio  $\lambda_{opt}$  with its correspondent maximum power coefficient  $C_{p,opt}$  (typ. 0.4-0.5), so the mechanical power has its maximum value  $P_{m,opt}$  for that wind speed. From (1) and (2), the following equations can be obtained:

$$\Omega_{opt} = \lambda_{opt} v / R \quad (4)$$

$$P_{m,opt} = \frac{1}{2} \rho A C_{p,opt} v^3 = \frac{1}{2} \rho A R^3 \frac{C_{p,max}}{\lambda_{opt}^3} \Omega_{opt}^3 \quad (5)$$

This last equation establishes the well-known cubic relationship between rotational speed and mechanical power used to maximize wind power production (see Fig. 3).

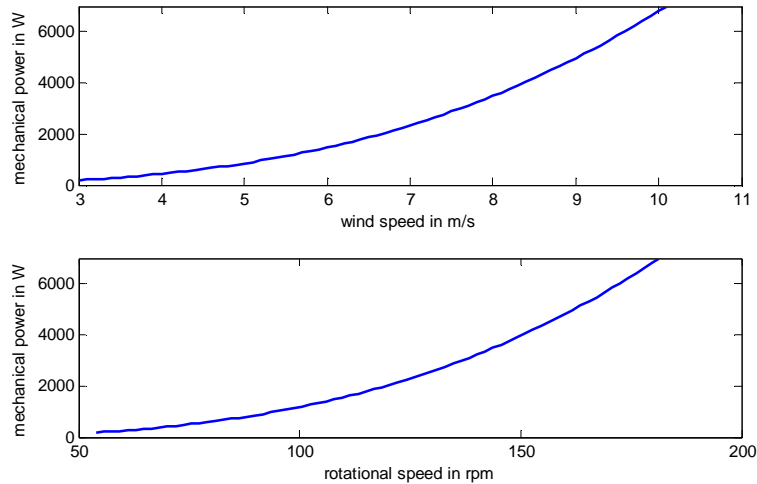


Fig. 3: Maximum mechanical power curves

### ***Permanent magnet synchronous generator (PMSG)***

Permanent magnet synchronous generators (PMSG's) are typically used in small wind turbines because due to several reason (high efficiency, gearless, simple control...).

A simple steady estate model has been chosen in order to evaluate the behaviour of this kind of machines [13], with the following assumptions:

- Saturation is neglected;
- Internal voltage or back emf is sinusoidal (harmonic components are neglected);
- Eddy currents and hysteresis losses are negligible.

In this case, a single phase equivalent circuit, as shown in Fig. 4, can be used and the following equation determines the relationship between voltages and current:

$$\underline{E} = \underline{U} + (R_s + j\omega L_s) \underline{I} \quad (6)$$

Where  $\underline{E}$  is the internal voltage,  $\underline{U}$  is the voltage in the estátor,  $\underline{I}$  is the current delivered,  $R_s$  and  $L_s$  are the

steady state resistance and inductance, respectively, and  $\omega$  is the frequency of voltage and currents in the PMSG stator. In steady state, frequency is related to rotational speed by mean the equation:

$$\omega = p\Omega \quad (7)$$

Where  $p$  is the number of pole pairs of PMSG.

Finally, the magnetic flux is supposed to be constant, so a linear relationship between rotational speed and no-load voltage is considered, so:

$$E = K_e \Omega \quad (8)$$

Where  $K_e$  is the relationship between  $E$  and  $\Omega$ .

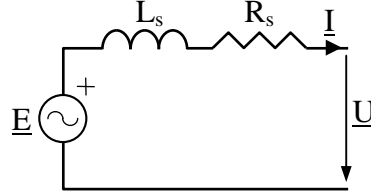


Fig. 4: PMSG equivalent circuit

### Wind energy conversion system (WECS)

In the SWTG considered in this paper, WECS is composed by a diode full-wave rectifier, dump load resistors and an inverter. By means of these elements, power extracted from wind can be delivered to the network in constant voltage and frequency conditions.

The rectifier has been modelled by mean the relationship between the input RMS voltage  $U$  and the DC voltage  $U_{DC}$ :

$$U_{DC} = 3\sqrt{3}\sqrt{2}/\pi U \quad (9)$$

In the rectifier input, voltage and current are in phase ( $\cos \phi = 1$ ), so the following equations can be written:

$$\underline{E} = U + (R_s + j\omega L_s) I \quad (10)$$

$$E^2 I^2 = P_m^2 + \omega^2 L_s^2 I^4 \quad (11)$$

A current source equivalent has been used to model the inverter. Its main parameters are the DC voltage range and the power curve. Both parameters affect to the wind energy production for a given rotor and PMSG. Power delivered by the inverter  $P$  in W can be written as:

$$P = 3U I \eta_{ele} \quad (12)$$

Where  $\eta_{ele}$  is the efficiency of WECS.

## Power Curve Optimization

### Inverter Power Curve

The inverter power curve relates an input variable (typ. rotor speed, DC voltage or frequency) with the desired inverter power output. Usually, frequency of the voltage in the PMSG side  $f_r$  or DC bus voltage  $U_{DC}$  is used as input for the mentioned curve. This curve, in commercial inverters, can be defined in several ways, typically by means a polyline [0 or by third degree polynomial [1,2,12]. In any case, inverter power curve must be designed to maximize the SWTG power production.

In certain inverters, inverter power curve is divided in two parts, the lower power part the power curve is linear and in the upper part the power curve tends to be cubic (see Fig. 5). The upper high power part can be expressed with a set of polyline vertices (frequency-power or DC voltage-power):

$$\{(X_0, P_0); (X_1, P_1); \dots; (X_n, P_n)\} \quad (13)$$

Where  $X$  is the input variable ( $f_r$  or  $U_{DC}$ ) and  $n$  the number of point. Another way to define the power curve is by means the coefficients ( $a_0, a_1, a_2$  and  $a_3$ ) of a third degree polynomial expression:

$$P_{pol} = a_0 + a_1 X + a_2 X^2 + a_3 X^3 \quad (14)$$

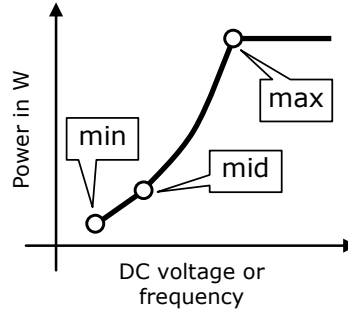


Fig. 5: Inverter power curve definition

So, the main inverter settings are: inverter power curve defined with (13) or (14), a minimum DC voltage to start generation ( $U_{DC,min}$ ), a middle DC voltage or power as a limit between the low and high power parts of the power curve parts and a maximum DC voltage or power when the power is fixed at the nominal value.

For the shake of simplicity lower part of the curve has been not taken into account in this paper.

### Power Curve Optimization

In large WTG's, an optimum power curve, as shown in (5), is usually used to define desired power production. In this case, the inverter power curve can be defined as:

$$P_{opt} = K_{opt} \omega^3 \quad (15)$$

Where:

$$K_{opt} = \frac{1}{2} \rho A R^3 \frac{C_{p,max}}{p^3 \lambda_{opt}^3} \eta_{mec} \eta_{ele} \quad (16)$$

And  $P_{opt}$  is the optimum power delivered by the WTG. A similar equation can be found to relate power and DC voltage.

In SWTG, the cubic equation shown in (15) could not be the most appropriate due to the high impedance values of PMSG. The maximum mechanical power for a given wind speed could not imply the maximum electrical power. In this paper, the maximum electrical power  $P_{max}$  delivered by the SWTG is obtained by using (3), (7), (8), (10), (12) and (15) with an algorithm of maximization [0 in Matlab (Mathworks Inc., Natick, US). The result is shown in Fig. 6, where the curve called "opt." is that obtained from (15), and the curve called "max." is obtained with the optimization process mentioned above. Furthermore, the maximum wind speed  $v_{lim}$  at which the power delivered by the inverter can have the values given in (15) is calculated with the equation:

$$v_{lim} = K_e \sqrt{\frac{\lambda_{opt}}{R A p C_p \eta_{mec}} \frac{1}{3 L_s p}} \quad (17)$$

For the data shown in TABLE I, this value is 10.23 m/s which implies an electrical power of approx. 4855W that is lower than the nominal one (see Fig. 6).

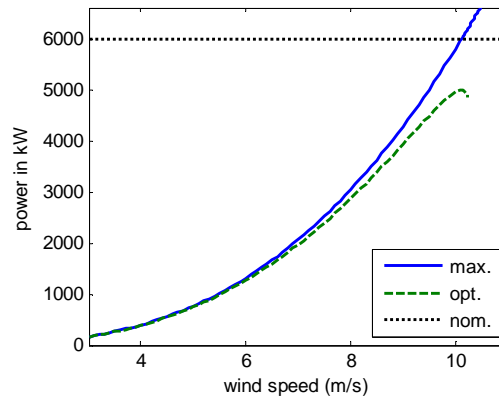


Fig. 6: Maximum power and optimum power curves

### Power Curve Calculation: polynomial approximation

In the previous paragraphs, the maximum achievable power  $P_{\max}$  has been obtained. This power can be used as reference for the inverter, so the maximum energy from wind can be obtained. To be implemented in the inverter, this curve can be approximated by a third degree polynomial equation, as that shown in (14). In order to be implemented in the inverter, the input values must be the frequency  $f_r$  or the DC voltage  $U_{DC}$ . By means a least-squared based algorithm, the approximated polynomial curves can be obtained, as shown Fig. 7. The different between the curves is quasi negligible, so the power achieved by the SWTG with this inverter power curve must be very close to the maximum one. For this reason, the polynomial approximation is taken as reference in this paper.

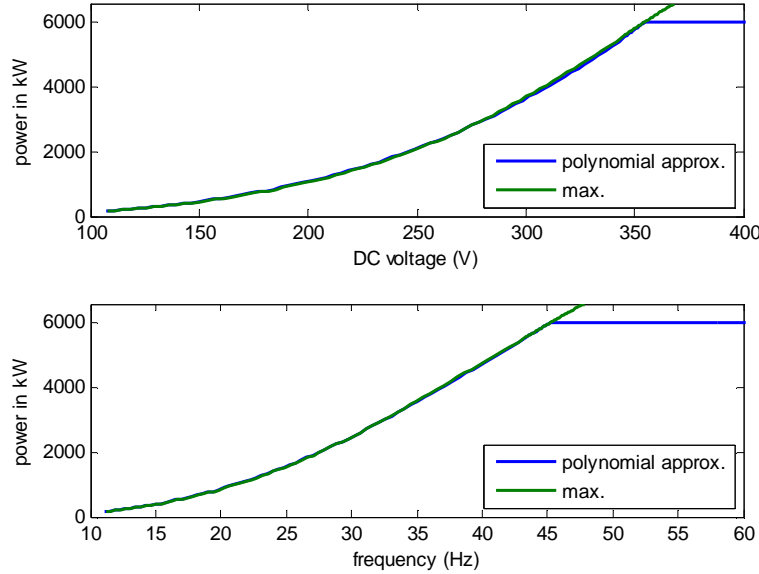


Fig. 7: Maximum power curve and its polynomial approximation.

### Power Curve Calculation: Polyline approximation

In many commercial inverters for WECS, the power curve must be defined as a set of vertices of a polyline, being the difference between different manufacturers the number of vertices allowed. The way to obtain these points and the size of the set determine the WECS behavior. In this paper, an optimization method to obtain these points is presented. The process to obtain the optimum set of  $(X_i, P_i)$  vertices has the following restrictions:

- 1) All the vertices must form an increasing curve.

$$\begin{aligned} X_0 < X_1 < \dots < X_n \\ P_0 < P_1 < \dots < P_n \end{aligned} \quad (18)$$

- 2) Power associated to the last vertex must be the nominal one  $P_{\text{nom}}$ .

$$P_n = P_{\text{nom}} \quad (19)$$

- 3) The maximum distance  $d_i$  between each segment formed by the vertices  $\{(X_i, P_i); (X_{i+1}, P_{i+1})\}$  and the  $P_{\max}$  curve must have a value lower than a given value  $-d_{\min}$  (see Fig. 8).

Finally, the polyline formed by the vertices  $(X_i, P_i)$  is approximated to the maximum power curve by a least-squares based algorithm with the mentioned above restrictions.

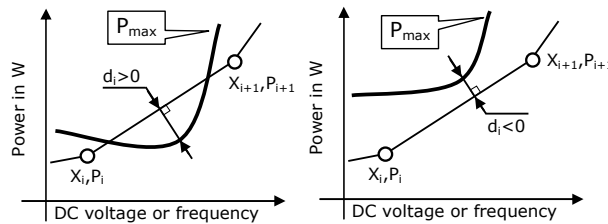


Fig. 8: Distance between  $P_{\max}$  curve and the polyline defined by  $(X_i, P_i)$  points.

The resulting vertices, i.e., the polyline approximation of  $P_{\max}$  curve, is shown in Fig. 9 for a set size of two vertices and using DC  $U_{DC}$  voltage as input. Similar results can be obtained using frequency  $f_r$  as input.

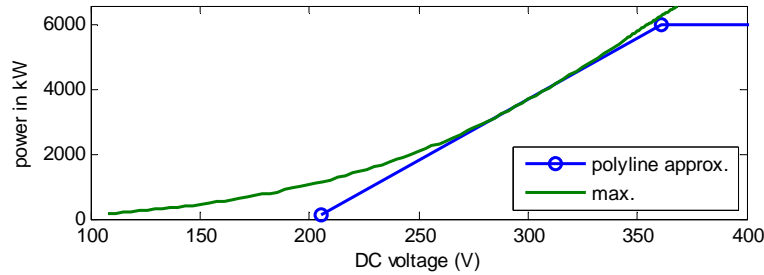


Fig. 9: Maximum power curve and its approximation by means a polyline defined by two vertices.

## Results

Once the inverter power curves have been obtained (polynomial and polyline approximation), they must be tested comparing their wind energy production in different wind regimes. For this purpose, a probability distribution of Rayleigh has been used for wind speed [16].

Results are shown in TABLE II, where the resulting mean power production has been obtained for different mean wind speeds (3-9 m/s), using the polynomial approximation and the polyline approximation (with 2, 3 and 4 vertices). The difference between the mean power obtained with the polyline approximation of the inverter power curve and the mean power obtained with the polynomial approximation one is also shown in the table. The difference between them is only significant when the wind speed is low (capacity factor  $< 0.2$ ) and the number of vertices (N. points) used in the polyline approximation is two. In Fig. 10 the mean power ratio (mean power obtained with the polyline approx. / mean power obtained with the polynomial approx.) is presented when the number of vertices for the polyline approximation is only two.

TABLE II. Mean power results

Mean Wind Speed (m/s)	Polynomial approx. Mean Power (W)	Polyline approx.					
		N. points: 2		N. points: 3		N. points: 4	
		Mean Power (W)	Diff.	Mean Power (W)	Diff.	Mean Power (W)	Diff.
3	278,48	252,78	-9,23%	277,75	-0,26%	278,48	0,00%
4	680,63	653,53	-3,98%	679,41	-0,18%	680,63	0,00%
5	1.235,16	1.211,40	-1,92%	1.233,72	-0,12%	1.235,16	0,00%
6	1.845,49	1.825,75	-1,07%	1.844,07	-0,08%	1.845,49	0,00%
7	2.415,04	2.398,83	-0,67%	2.413,74	-0,05%	2.415,04	0,00%
8	2.879,37	2.866,00	-0,46%	2.878,21	-0,04%	2.879,37	0,00%
9	3.208,93	3.197,81	-0,35%	3.207,92	-0,03%	3.208,93	0,00%

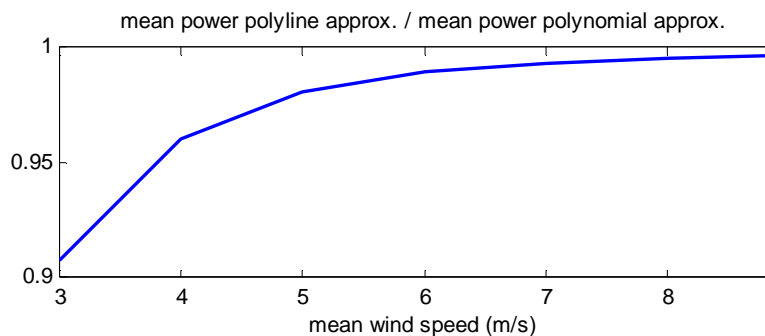


Fig. 10: Mean power ratio

Apart from energy production, the working limits of power curve are an important issue because they affect to the WECS dynamic behavior (see TABLE III). The relationship between maximum and minimum values of DC voltages or frequencies of the power curve is a measure of available capability of the inverter to follow the power curve. In this case, the number of points of the polyline approximation is three; its behavior is very close to that with the polynomial approximation.

TABLE III. Voltage and frequency limits

	Polynomial approx.	Polyline approx.		
		N. points: 2	N. points: 3	N. points: 4
Vdc Min (V)	103	205	120	106
Vdc Max (V)	356	361	358	361
Vdc Max/Min	3,5	1,8	3,0	3,4
fr Min (Hz)	11,5	20,5	13,0	11,5
fr Max (Hz)	45,2	45,4	45,4	45,4
fr Max/Min	3,9	2,2	3,5	3,9

Another question that must be considered is how the WECS behavior is affected by the variation of PMSG parameters. In this case, variations of  $\pm 10\%$  and  $\pm 20\%$  have been supposed in the steady state inductance  $L_s$  and in the internal voltage  $E$  (derived from the no-load voltage). The inverter power curves (fr-P and  $V_{DC}$ -P) obtained by polyline approximation (n. points: 2) are compared to the maximum available power  $P_{max}$  at different  $L_s$  and  $E$  values.

In Fig. 11 and Fig. 12 (see left) the resulting maximum power curves and the polyline power curve at different  $L_s$  and  $E$  values are shown. In these cases, DC voltage is used as input for the power curve. In Fig. 11 and Fig. 12 (see right) a similar analysis is shown but using frequency as input. Main conclusions are:

- Polyline inverter power curve must be designed to manage PMSG parameters variations, because when  $P_{max}$  curve is lower than the polyline one, unstable situations can appear during SWTG operation.
- Variations on steady state inductance  $L_s$  are more significant when frequency is used as input in the power curve.
- Variations on open voltage  $E$  are more significant when DC voltage is used as input in the power curve. In this case, to avoid unstable conditions the polyline power curve must be gently displaced to higher voltages.

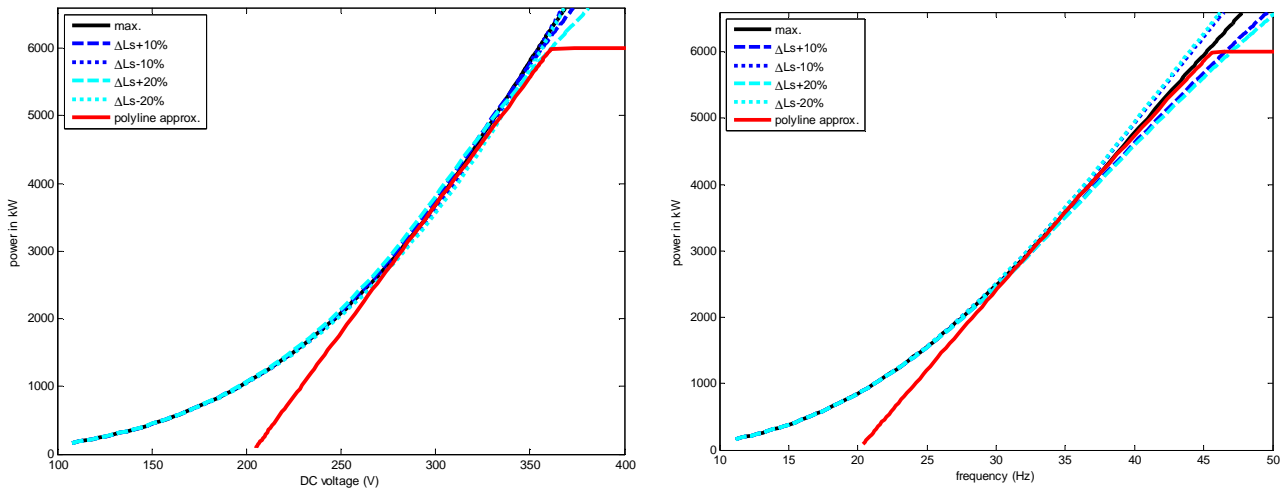


Fig. 11: Maximum power with  $L_s$  variations and polynomial power curve with DC voltage (right) and frequency (left) in the X-axis.



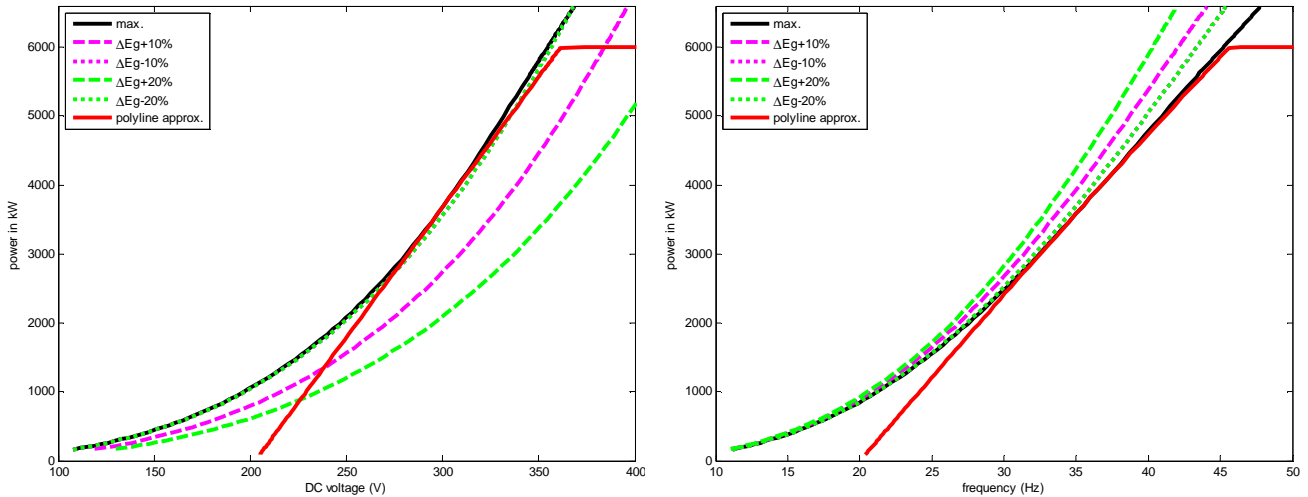


Fig. 12: Maximum power with E variations and polynomial power curve with DC voltage (right) and frequency (left) in the X-axis

## Conclusions

In this paper a SWTG with a WECS formed by a rectifier, dump load resistors and an inverter is considered.

The paper is focused in the way that the power curve is implemented in the inverter in order to optimize the wind energy production. The following situations have been analyzed:

- Power curve is implemented by means a third degree polynomial function;
- Power curve is implemented by means a set of point, i.e., a polyline approximation;
- Voltage in the DC bus is used as input for the power curve;
- The frequency in the PMSG side is used as input.

The polynomial approximation has been calculated using a least-squared based algorithm. For the polyline approximation a optimization method is proposed that takes into account the distance between the maximum power curve and the proposed polyline approximation.

Using a third degree polynomial function a good agreement between maximum power curve and the approximated power curve is obtained. However, many commercial inverters do not use this method to implement its power curve and the curve is implemented by means a polyline.

When approximated power curves to be implemented in the inverter have been obtained different analysis have been done. Firstly, the impact of the number of vertices used to define the polyline is evaluated. When this number is greater than two a good approximation power curve is obtained.

The margin of input values (frequency or DC voltage) is also evaluated as function of the number of vertices. In this case the behavior of the polynomial approximation and the polyline one are very close when the number of vertices is greater than three.

Finally, the impact of variation in the PMSG parameters is considered. This impact depends on which input the inverter power curve has. When the input is the DC voltage, the inverter curve must be gently displaced to avoid intercept the maximum power curve; which may derive in unstable behavior. So, using frequency as input for the inverter power curve a more stable can be expected against different PMSG parameters.

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